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### Plant and Soil

DOI:

[10.1007/s11104-016-2883-4](https://doi.org/10.1007/s11104-016-2883-4)

Published: 01/09/2016

Peer reviewed version

[Cyswllt i'r cyhoeddiad / Link to publication](#)

*Dyfyniad o'r fersiwn a gyhoeddwyd / Citation for published version (APA):*

Heppell, J., Payvandi, S., Talboys, P., Zygalkis, K. C., Langton, D., Sylvester-Bradley, R., Edwards, A. C., Walker, R., Withers, P., Jones, D., & Roose, T. (2016). Use of a coupled soil-root-leaf model to optimise phosphate fertiliser use efficiency in barley. *Plant and Soil*, 406(1-2), 341-357. <https://doi.org/10.1007/s11104-016-2883-4>

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# Use of a coupled soil-root-leaf model to optimise phosphate fertiliser use efficiency in barley

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## Abstract: (200/200)

*Aims* Phosphorus (P) is an essential nutrient necessary for maintaining crop growth, however, it's often used inefficiently within agroecosystems, driving industry to find new ways to deliver P to crops sustainably. We ~~consider a precision agriculture approach which aim to~~ combine traditional soil and crop measurements with climate-driven mathematical models, ~~that can to give insight into~~ optimise the timing and placement of fertiliser applications.

*Methods* The whole plant crop model combines an above-ground leaf model with an existing spatially explicit below-ground root-soil model to estimate plant P uptake and ~~above ground dry leaf~~ mass. We let P-dependent photosynthesis estimate carbon (C) mass, which in conjunction with temperature sets the root-growth-rate.

*Results* The addition of the leaf model achieved a better estimate of two sets of barley field trial data for ~~leaf mass and~~ plant P uptake, compared with just the root-soil model alone. Furthermore,

23 discrete fertiliser placement increases plant P uptake by up to 10% in comparison to incorporating  
24 fertiliser.

25 *Conclusions* By capturing essential plant processes we are able to accurately simulate P and C use  
26 and water and P movement during a cropping season. The powerful combination of mechanistic  
27 modelling and experimental data allows physiological processes to be quantified accurately and  
28 useful agricultural predictions for site specific locations [to be made](#).

29 **Keywords** Mathematical modelling, phosphate, phosphorus, fertiliser strategy, barley field study,  
30 above and below ground

## 31 **Introduction**

32 The world-wide production of food has increased due to the demands of an ever expanding global  
33 human population (Brown, 2012). Due to the lack of land available for agricultural expansion, there  
34 is a need to increase [crop](#) yields sustainably by manipulating the existing environment in which crops  
35 are grown, and breeding more resource efficient crops. Resource management for arable farming  
36 systems is critical to the survival of the human population and large amounts of money and time are  
37 needed to elicit the appropriate improvements (Conway and Barbier, 1990).

38 [Phosphorus \(P\) is one of the essential nutrients required for plant growth and plays an important](#)  
39 [role in photosynthesis, respiration, and seed and fruit production.](#)

40 We are interested in how crops grow and survive in low P environments and how fertiliser and soil  
41 cultivation methods are influencing crop performance. A number of studies have considered the  
42 response of adding different amounts and rates of fertiliser P; in some soils large effects are seen  
43 whereas no effect is seen in others (Bolland and Baker, 1998; Kuchenbuch and Buczko, 2011;  
44 Valkama et al., 2011). There are many ways one could apply P to soils; for example incorporating  
45 (also known as broadcasting, involves an even spreading of P on top of the soil), placing (also known  
46 as banding, involves injecting P into the soil nearer the rooting zone either in row or between rows)

47 or as a coating on seeds. Studies have shown that injecting fertiliser into the soil nearer to the root  
48 zone (placing) increases plant P uptake compared to incorporated P (Randall and Hoeft, 1988; Lohry,  
49 1998; Owusu-Gyimah *et al.*, 2013). In addition, studies have been conducted to estimate the  
50 differences in soil cultivation methods on plant P uptake; for example, conventional plough *versus*  
51 minimum tillage (also considering gene variation, George *et al.*, 2011). The idea behind ploughing is  
52 to turn over or mix the top 25 cm of soil to loosen the soil for seeding, bury any existing crop  
53 residues or weeds, and to provide a good distribution of nutrients for the coming crop. This is in  
54 contrast to minimum tillage which enhances topsoil stability against erosion, retains moisture and  
55 reduces crop establishment costs, but segregates P content with depth and can leave 30% of crop  
56 residue on the soil surface.

57 Due to the rising cost of fertilisers and agricultural machinery, crop production has become a multi-  
58 objective optimisation problem to minimise multiple costs while trying to maximise the crop yield  
59 and environmental impact of fertilisers. This is a complex problem due to varying climatic conditions,  
60 an abundance of technological machines, and availability of more data concerning the states of  
61 fields than ever before. Precision agriculture is an emerging field involved with combining the  
62 newest technologies to the farming industry, ranging from unmanned drone maps of fields to  
63 computer-assisted tractors (Blackmore, 2014). This new technology is enabling automated real time  
64 decision making, applying the most effective treatment to crops at the best time for the best price.  
65 Mathematical models, supported by experimental data, are needed to help predict best decisions in  
66 the short term, and also strategically, to optimise between possible future options. Whilst such  
67 models are ~~seldom-not always commercially used~~~~employed at present~~, their potential capabilities  
68 are attractive, given that field-scale experiments are both costly and time-consuming, and  
69 integration and dissemination of their empirical results is challenging (Selmants and Hart, 2010;  
70 Jeuffroy *et al.*, 2012; Sylvester-Bradley, 1991).

71 A plethora of models exist that describe the processes involved in plant growth and the behaviour of  
72 nutrients and water in the soil. Each model has its own unique assumptions and is generally targeted  
73 at specific scientific problems within the area of agriculture. For example, Greenwood *et al.* (2001)  
74 developed a dynamic model (PHOSMOD) for the effects of soil P and fertiliser P on crop growth, P  
75 uptake and soil P in arable cropping;- Jones *et al.* (2003) describe a decision support system for  
76 agrotechnology transfer (DSSAT) which focuses on average plant-environment interactions; and  
77 Keating *et al.* (2003) review an agricultural production systems simulation (APSIM) developed in  
78 CISRO, Australia which deals with water, N, P, pH, erosion and management issues. At the beginning  
79 of the 21<sup>st</sup> century, modelling 3D architectures of plant roots (RootBox, ROOTMAP, SimRoot,  
80 RootTyp, SPACSYS, R-SWMS) has become popular (Dunbabin *et al.*, 2013). In addition, two research  
81 groups that model above ground 3D plant structures, Prunsinkiewicz Algorithmic Botany group at  
82 the University of Calgary and the Andrieu group (ADEL-wheat model), both use L systems to simulate  
83 the above ground structure of wheat plants. L systems, introduced by Lindenmayer in 1968,  
84 represent a string of production rules that are used to create geometric structures, ideal for plant  
85 development. However all these models do not describe the root-soil interaction explicitly and do  
86 not fully integrate functions that occur above ground with ones that occur below ground. Therefore  
87 plants of the same genotype are represented alike and phenotypic differences cannot be observed.  
88 We hope to address some of these problems by creating a model that links the above and below  
89 ground processes in such a way that they rely on one another. Our whole crop model is based on a  
90 below ground plant-soil interaction model (Roose and Fowler, 2004b; Heppell *et al.*, 2015) coupled  
91 with an above ground leaf growth model based on the seminal work of Thornley (1995).

92 Here we describe a whole crop model that includes a below-ground root model and an above-  
93 ground leaf model and which is validated against experimental data on barley with a varying P  
94 fertiliser scenario analysis. The development of the model is seen as a step-change in our  
95 computational capability to help predict soil P supply, crop P uptake patterns and fertilizer  
96 requirements.

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## 97 Materials and Methods

### 98 Experimental data

99 Two barley field trial data sets are used, consisting of ~~leaf mass~~<sup>above ground dry mass</sup> and plant P  
100 uptake values at different growth stages (GS31, GS45 and GS91 for spring barley; GS39 and GS92 for  
101 winter barley). The experimental data includes different rates of P application (0, 5, 10, 20, 30, 60, 90  
102 kg P ha<sup>-1</sup> for spring barley; 0 15, 30, 60, 90, 120 kg P ha<sup>-1</sup> for winter barley) and both sites were  
103 classified with an Olsen P index 1 soil. -The protocol for this is described in Heppell *et al.* (2015). In  
104 addition, we use the climate data, from the UK Met office Integrated Data Archive System (MIDAS),  
105 to accompany the spring barley (Inverurie, Scotland) and winter barley (Cambridge, England) data  
106 sets for the specific fields in the trial. The climate data consists of daily values for mean temperature  
107 (°C), rainfall (mm), wind speed (m s<sup>-1</sup>) and humidity (%).

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### 108 Modelling the whole crop

109 In this paper we extend a root-soil model (Roose and Fowler, 2004b; Heppell *et al.*, 2015) which  
110 estimates plant P uptake, with an above ground ~~leaf~~<sup>above ground dry</sup> model which estimates ~~leaf~~<sup>leaf</sup>  
111 mass (based on Thornley, 1995), to produce a whole crop model. We first describe the root-soil  
112 model (hereafter called the root model), followed by the leaf model and then our coupling process  
113 to create a whole crop model.

### 114 Root and soil model

115 To model the root system we follow the same approach as described in Roose *et al.* (2004b) and  
116 Heppell *et al.* (2015) by modelling two orders of root branches only (main and first order branches).  
117 First order roots branch off the main order roots at a given density ( $\psi_1$ ), branching angle ( $\theta$ ), and  
118 each order of roots has a given maximum length and radius ( $L_0$ ,  $L_1$  and  $a$ ,  $a_1$  for main and first order  
119 roots, respectively). As in Roose *et al.* (2004b) and Heppell *et al.* (2015) we let the root growth slow

120 down as the root becomes longer. Following Heppell *et al.* (2015) we also let the root growth rate ( $r$ )  
 121 be dependent upon [air](#) temperature  $T$ , [we detained from the MIDAS database.](#)

122 Eqn. 1 
$$\frac{\partial l_i}{\partial t} = r(T(t)) \left(1 - \frac{l_i}{L_i}\right),$$

123 where  $l_i$  is the current length of an order  $i$  root and  $L_i$  is the maximum length of an order  $i$  root.

124 The root-soil model is described by the following two equations for water saturation [\(S\)](#) (Eqn. 2) and  
 125 P (Eqn. 3) concentration [\(c\)](#) respectively,

126 Eqn. 2 
$$\phi \frac{\partial S}{\partial t} = \nabla \cdot [D_0 D(S) \nabla S - K_S k(S) \hat{\mathbf{k}}] - F_w(S, z, t),$$

127 Eqn. 3 
$$\frac{\partial}{\partial t} [(b + \phi S)c] + \nabla \cdot [c\mathbf{u}] = \nabla \cdot [D_f \phi^d S^d \nabla c] - F(c, z, t),$$

128 where the water flux in the soil,  $\mathbf{u}$ , is given by Darcy's law,

129 Eqn. 4 
$$\mathbf{u} = -D_0 D(S) \nabla S + K_S k(S) \hat{\mathbf{k}}.$$

130 In the above equations  $S$  is the relative water saturation given by  $S = \phi_1 / \phi$ ,  $\phi_1$  is the volumetric  
 131 water content, and  $\phi$  is the porosity of the soil.  $D_0$  ( $\text{cm}^2 \text{ day}^{-1}$ ) and  $K_S$  ( $\text{cm day}^{-1}$ ) are the parameters  
 132 for water 'diffusivity' and hydraulic conductivity, respectively (Van Genuchten, 1980).  $D(S)$  and  
 133  $K(S)$  characterize reduction in water 'diffusivity' and hydraulic conductivity in response to the  
 134 relative water saturation decrease, where the functional forms for partially saturated soil are given  
 135 by Van Genuchten (1980).  $\hat{\mathbf{k}}$  is the vector pointing vertically downwards from the soil surface and  $F_w$   
 136 is the water uptake by the plant root system per unit volume of soil as given by Roose and Fowler  
 137 (2004a).

138 For the total P conservation (Eqn. 3),  $c$  is the P concentration in soil pore water,  $b$  is the soil buffer  
 139 power characterising the amount of P bound to the soil particle surfaces,  $D_f$  is the P diffusivity in  
 140 free water and  $d$  is an impedance factor;  $1 \leq d \leq 3$  (Barber, 1984; Nye and Tinker, 1977).  $F(c, S, t)$   
 141 describes the rate of plant P uptake by a root branching structure (Roose *et al.*, 2001). Both  $F_w$  and  $F$

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are affected by the spatially and temporally evolving root structure. Water is only taken up by the main order roots [and the small region of first order roots near the branch point](#) while P is taken up by all roots; see Roose and Fowler (2004b) for details of the derivation. The equation for  $F_w$  is given by,

Eqn. 5 
$$F_w = \frac{2\pi a_1 k_r + (2\pi a_1 k_r k_z)^{\frac{1}{2}} \psi_1(z)}{\pi(a + L_1 \cos \theta)^2} [-p_c f(S) - p_r],$$

where  $\psi_1$  is the density of first order roots on the main order roots,  $a_1$  is the first order root radius,  $a$  is the main order root radius,  $L_1$  is the maximum length of the first order branches,  $\theta$  is the angle between the main root and the first order branches,  $k_r$  is the root radial water conductivity parameter ( $\text{m s}^{-1} \text{Pa}^{-1}$ ),  $k_z$  is the root axial hydraulic conductivity calculated using Poiseuille law ( $\text{m}^4 \text{Pa}^{-1} \text{s}^{-1}$ ),  $p_c$  (Pa) is a characteristic suction pressure determined from experimental data for different types of soil,  $f(S) = (s^{-1/m} - 1)^{1-m}$ , where  $m$  is the Van Genuchten soil suction parameter (where  $0 < m < 1$ ), and  $p_r$  is the root internal xylem pressure (Pa).

Root internal xylem pressure ( $p_r$ ) is calculated by balancing radial and axial fluid fluxes inside the root, i.e. after Roose and Fowler (2004a) we have,

Eqn. 6 
$$2\pi a k_r (-p_c f(S) - p_r) = -k_z \frac{\partial^2 p_r}{\partial z^2},$$

with two boundary conditions; an impermeable root tip (Eqn. 7) and a root internal pressure ( $P$ ) at the base of the zero order root (Eqn. 8),

Eqn. 7 
$$\frac{\partial p_r}{\partial z} = 0 \text{ at } z = L,$$

Eqn. 8 
$$p_r = P \text{ at } z = 0,$$

where  $P$  is a function of temperature ( $T$ ), humidity ( $H$ ) and a base line pressure ( $p_r^0$ ) for fitting parameters  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$  (see Heppell *et al.*, 2014 for the procedure to estimate them), i.e.



163 Eqn. 9  $P = (p_r^0 + \lambda_3) + \lambda_1 T + \lambda_2 H.$

164 The rate of plant P uptake is given by,

165 Eqn. 10 
$$F(c, z, t) = \frac{F_0 + F_1}{\pi(a + L_1 \cos \theta)^2},$$

166 where  $F_0$  and  $F_1$  are the uptake rates for zero and first order roots; [see derived in](#) Roose *et al.*  
167 (2004b) [for derivation](#).

168 The boundary conditions to accompany Equations 1 and 2 include a soil surface boundary condition  
169 for water,

170 Eqn. 11 
$$-D_o D(S) \frac{\partial S}{\partial z} + K_S k(S) = W_{dim} \text{ at } z = 0.$$

171  $W_{dim}$  (the flux of water into the soil) is dependent upon rainfall ( $R$ ), humidity ( $H$ ), temperature ( $T$ ),  
172 wind speed ( $WS$ ) and a constant ( $E$ ) which sets a base line flux i.e.

173 Eqn. 12 
$$W_{dim} = \delta R + \alpha H + \beta T + \gamma WS + E,$$

174 for fitting parameters  $\delta$ ,  $\alpha$ ,  $\beta$  and  $\gamma$  (see Heppell *et al.*, 2014 for how these values were estimated).

175 In addition, we have a [zero flux](#) boundary condition for the concentration of P ( $c$ ) at the soil surface,

176 Eqn. 13 
$$-D_f \phi^d S^d \frac{\partial c}{\partial z} + W_{dim} c = 0 \text{ at } z = 0, \text{ for } t > 0.$$

177 We set a zero flux at the bottom of the soil ( $l_w$ ) for both P and water,

178 Eqn. 14 
$$-D_o D(S) \frac{\partial S}{\partial z} + K_S k(S) = 0 \text{ at } z = l_w,$$

179 Eqn. 15 
$$-D_f \phi^d S^d \frac{\partial c}{\partial z} = 0 \text{ at } z = l_w.$$

180 The initial state of P concentration and water saturation in the soil is given, where possible, by the  
181 initial soil data for the spring and winter barley experimental sites. A uniform water saturation  
182 profile is initially set at  $S = 0.3$  for the two experimental sites; however for the initial P

183 concentration ( $c_0(z)$ ) we consider two different cases; (1) a uniform concentration and (2) an  
 184 exponentially decaying concentration:

185 Eqn. 16

$$\begin{aligned} (1) \quad c_0(z) &= c_A & \text{at } t = 0, \forall z \\ (2) \quad c_0(z) &= A_1 e^{-B_1 z} & \text{at } t = 0, \forall z' \end{aligned}$$

186 where  $c_A$  is set to 16 mg P l<sup>-1</sup>,  $A_1$  is the P concentration at the top of the soil (23 mg P L<sup>-1</sup>) and  $B_1$  is  
 187 the strength of the decay in the concentration of P (0.345). The initial P concentration values ( $C_A$ ,  
 188  $A_1$  and  $B_1$ ) come from a best fit to the data sets in Heppell et al., (2015) and are both classified as an  
 189 Olsen P index 1 soil (Defra, 2010). To reflect the different fertiliser scenarios being used at each field  
 190 site a set amount of P ( $P_1$ ) (0-120 kg P ha<sup>-1</sup>) was either applied at the surface ( $z = 0$ ) (P broadcast) or  
 191 at a set depth below the soil ( $D_1$ ) (P placement).

192 Eqn. 17

$$\begin{aligned} &c = c_0(z) + H(z), \\ \text{(broadcast)} \quad &H(z) = P_1 & \text{at } t = 0, \quad z = 0 \\ \text{(placement)} \quad &H(z) = P_1 & \text{at } t = 0, \quad z = D_1 \\ \text{(else)} \quad &H(z) = 0 & \text{at } t = 0 \quad \forall z \end{aligned}$$

193 [With the soil P profile initialised \(Equations 16 and 17\) we are able to estimate \(belowground only\)](#)  
 194 [the water and P concentrations in the soil by solving Equations 1-15, as in Heppell \*et al.\*, 2014.](#)

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## 195 Leaf growth model

196 We have altered a compartmental model developed by Thornley (1995) to describe leaf mass [\(a](#)  
 197 [proxy for above ground dry mass\)](#)  $M_L$  (kgL), leaf C  $M_C$  (kgC) and leaf P  $M_P$  (kgP) as well as the  
 198 concentration of free C  $[C] = M_C/M_L$  (kgC kgL<sup>-1</sup>) and free P  $[P] = M_P/M_L$  (kgP kgL<sup>-1</sup>) dynamics  
 199 within the leaves. The leaf model takes into account non-linear dynamics of formation of leaf litter  
 200 and leaf self-shading. Additionally we have made photosynthesis dependent upon P content in the  
 201 plant (Foyer and Spencer, 1986, Wissuwa *et al.*, 2005) and we have altered the leaf growth term,  $G_{sh}$ ,  
 202 [which was dependent on \[C\] and \[P\],](#) to also depend upon ~~the~~ air temperature ( $A_T$ ), ~~for the~~ winter  
 203 ~~barley, but not for spring barley), as well as [C] and [P].~~ [We do not let air temperature affect spring](#)  
 204 [barley as the growing season is much shorter compared to winter barley and it appeared not to be](#)

needed for a good fit to the experimental data. The governing equations are given below and are represented in a flow diagram on Figure 1, i.e., we have

$$\text{Eqn. 18} \quad \frac{\partial M_L}{\partial t} = \overbrace{\widehat{G}_{sh}}^{\text{Leaf growth rate}} - \overbrace{\frac{K_{litt}}{1 + \frac{K_{mlitt}}{M_L}} M_L}^{\text{Leaf metabolism/litter}},$$

$$\text{Eqn. 19} \quad \frac{\partial M_C}{\partial t} = \overbrace{\varepsilon k_1 [P]}^{\text{Production of C from photosynthesis}} - \overbrace{f_c \widehat{G}_{sh}}^{\text{Use of C for L growth}} - \overbrace{\beta_c [C]}^{\text{Output of C to phloem}},$$

$$\text{Eqn. 20} \quad \frac{\partial M_P}{\partial t} = \overbrace{-f_p \widehat{G}_{sh}}^{\text{Use of P for L growth}} + \overbrace{F(c, z, t)}^{\text{Input of P from xylem}} - \overbrace{\beta_p [P]}^{\text{Output of P to phloem}} - \overbrace{k_p \varepsilon [P] k_1}^{\text{Use of P to produce C}},$$

where,

$$\text{Eqn. 21} \quad G_{sh} = k_G M_L [C] [P] \frac{A_T^{s_1}}{s_2^{s_1} + A_T^{s_1}},$$

$$\text{Eqn. 22} \quad \varepsilon = \frac{k_C M_L}{\left(1 + \frac{M_L}{k_M}\right) \left(1 + \frac{[C]}{J_C}\right)},$$

where  $k_g$  is the leaf growth rate,  $K_{litt}$  is the litter rate,  $K_{mlitt}$  is the litter Michaelis-Menten constant,  $K_C$  is the photosynthesis rate,  $k_M$  is the constant accounting for the leaf self-shading,  $J_C$  is the C product inhibition constant,  $f_c$  is the fraction of total C used for leaf growth,  $f_p$  is the fraction of total P used for leaf growth,  $k_1$  is the amount of P used for photosynthesis,  $k_p k_1$  is the P loss due to photosynthesis,  $\beta_c$  is the rate of C output from the xylem to the phloem,  $\beta_p$  is the rate of P output to the phloem,  $F(c, z, t)$  is the rate of P entry from the xylem (Eqn. 10) and  $s_1$  and  $s_2$  are fitting parameters. Initial values for the leaf ( $M_L$ ), C ( $M_C$ ) and P ( $M_P$ ) mass are  $1 \times 10^{-4}$ , 0 and  $1 \times 10^{-7}$  kg respectively.

## Whole crop model

In order to provide feedback between the root model and leaf model, we allow C mass to affect the root growth rate. Increasing C mass will increase root growth which in turn will increase plant P

uptake. Through the process of photosynthesis, increasing plant P uptake will also increase C mass, thus creating a positive feedback loop.

The order  $i$  root growth rate is now dependent on C as well as temperature, therefore we replace Eqn. 1 with,

Eqn. 23 
$$\frac{\partial l_i}{\partial t} = r(T, C) \left(1 - \frac{l_i}{L_i}\right),$$

where the rate of growth  $r(T, C)$  is given by a function of temperature multiplied by a function of C ( $r(T, C) = f(C)g(T)$ ),

Eqn. 24 
$$f(C) = \frac{\alpha_c M_c}{\gamma_c + M_c},$$

Eqn. 25 
$$g(T) = \begin{cases} 0 & T \leq 5^\circ C \\ A(T - 5) & T > 5^\circ C \end{cases}$$

where  $\gamma_c$  is the mass of C when the root system is at half its maximum size,  $\alpha_c$  is the strength of the C effect and  $A$  is a fitting parameter determining the strength of temperature dependence on root growth rate. Below critical temperature ( $5^\circ C$ ) there is no root growth and this reflects cold periods over the winter (Sylvester-Bradley *et al.*, 2008).

## Fitting process Calibration

The parameter list for the models above is given in Table 1. A subset of these parameters are fitted to the experimental data and their values can be seen in Table 2. To begin the fitting procedure calibration process, the leaf model is first fit against the experimental leaf-above ground dry mass data, by changing 4-6 parameters ( $\beta_c$ ,  $k_1$ ,  $f_c$ ,  $f_p$  for spring barley and in addition  $s_1$  and  $s_2$  for winter barley). In the leaf model only, we set the rate of P entry from the xylem ( $F(c, z, t)$ , Eqn. 10) proportional to the experimental plant P uptake to simulate a representative plant P root uptake. We then combine the models, i.e. let the rate of P entry from the xylem be estimated from the root model, and fit for the remaining parameters ( $\gamma_c$  and  $\alpha_c$ ).

246 During the ~~fitting process~~calibration step we minimise the sum of squares value between the  
247 plant P uptake and leaf above ground dry mass values against the experimental data values for ~~each~~  
248 control and maximum applied P scenario (0 and 90/120 kg P ha<sup>-1</sup> respectively). With the fitted  
249 parameters we then run the model for all applied P scenarios.

250 The differences between modelling spring barley and winter barley are the time they are grown for  
251 (151 and 313 days, respectively), the initial P profile in the soil (20 mg P l<sup>-1</sup> decay profile and 16 mg P  
252 l<sup>-1</sup> constant profile, respectively) and leaf growth dependence (also depending upon air temperature  
253 for winter barley).

## 254 Results

255 We compare two sets of barley field experimental data against the coupled model, the leaf model  
256 (where plant P uptake is given by experimental data) and the root model. The aim is to address the  
257 differences between the models and how well they fit the experimental field data for barley.

258 First we compare the values for plant P uptake between the root and coupled model for spring  
259 barley at three different growth stages, GS31, GS45 and GS91 for seven applied P rates (0, 5, 10, 20,  
260 30, 60 and 90 kg P ha<sup>-1</sup>; Figure 2). The coupled model estimates higher plant P uptake compared to  
261 the root model, better fitting the experimental data; staying within one standard deviation except at  
262 high applied P rates (30, 60 and 90 kg P ha<sup>-1</sup> at GS31, 20, 60 and 90 kg P ha<sup>-1</sup> at GS45 and 30 and 60  
263 kg P ha<sup>-1</sup> at GS91). The feedback effect within the coupled model enables the root structure to  
264 become larger than in the root model and therefore the roots explore more of the soil and hence  
265 achieve an increased plant P uptake ~~(Figure 8)~~. The final model estimate (GS91) is more accurate  
266 than the earliest (GS31) due to not capturing the effects of possible lateral root proliferation due to  
267 higher applied P rates (Drew, 1975). Early differences are averaged out as the root system grows.

268 When considering plant P uptake in winter barley, the coupled model behaves similarly to the root  
269 model (Figure 3). At GS92, both models under-predict plant P uptake for the same reasons as stated

270 in Heppell *et al.* (2015); the P profile is depleted which limits the amount of P available for uptake,  
271 and perhaps the total amount of P in the soil was different to that estimated by the one soil test for  
272 the whole site (Olsen P index 1). The effect of slow release P pools in the soil was not taken into  
273 consideration due to the fact experimental data for this phenomenon was not available.

274 By coupling the root model with the leaf model we are able to compare measured [leaf](#)-[above ground](#)  
275 [dry](#) mass values against the coupled and leaf model only for both spring barley (Figure 4) and winter  
276 barley (Figure 5) for different applied P rates. The coupled model accurately predicts [above ground](#)  
277 [dry](#)-[leaf](#) mass at GS91 for spring barley, however it estimates a more average value for earlier growth  
278 stages; not distinguishing any differences between applied P rates. The large errors bars in the  
279 experimental [above ground dry mass](#) [leaf mass](#) data are possibly due to field variation, making it  
280 hard to distinguish any differences between applied P rates, especially at later growth stages (the  
281 experimental differences are not statically significant). In addition, the variation in experimental  
282 plant P uptake values for GS31 is less than for GS91 (18% to 24%), implying little correlation between  
283 early and late plant P uptake (adjusted  $r^2=0.4$ ). For winter barley, the coupled model is able to match  
284 [above ground dry](#)-[leaf](#) mass at GS39, but vastly underestimates [leaf mass](#) at GS92 due to  
285 underestimating plant P uptake as mentioned above. The leaf model fits well across all scenarios for  
286 spring and winter barley as it takes the known plant P uptake from the experimental data as an input.

287 The leaf model component allows us to estimate P (Figure 6) and C mass (Figure 7) in the above  
288 ground tissue over the growing period of the crop. The estimated P mass is higher in the leaf model  
289 compared to the coupled model for both spring and winter barley. The estimated C mass is higher in  
290 the leaf model compared to the coupled model for winter barley, but the other way around for  
291 spring barley. In the winter barley case, the increased C and P masses in the leaf model are due to  
292 higher plant P uptake values (Figure 3 compared to Figure 2) resulting in a larger end [above ground](#)  
293 [dry](#)-[leaf](#) mass. For spring barley, C mass in the coupled model begins lower and ends higher  
294 compared to the leaf model because plant P uptake by the root system also begins lower and ends

295 higher (P uptake remains constant in the leaf model). The sudden decrease in C and P mass, for  
296 winter barley, around the 250 day mark is due to the enforced halting of the root growth rate.

297 The root growth rate is affected by C mass (spring barley) and also temperature (winter barley);  
298 therefore different final root lengths can be observed between model simulations (Figure 8). The  
299 leaf model created a longer root length compared to the coupled model in the winter barley  
300 scenario due to the early differences in C mass. For spring barley, the early C mass values for the  
301 coupled and leaf model were similar resulting in almost identical root growth rates and hence final  
302 root lengths. As C mass increases above a certain value any differences are masked when affecting  
303 the root growth rate. There was little difference in root length between the two different fertiliser  
304 applications (0 and 90/120 kg P ha<sup>-1</sup>), the largest being between the coupled model for winter barley  
305 GS92. Due to the small increase in plant P uptake between scenarios (0 and 120 kg P ha<sup>-1</sup>) there was  
306 little effect on increasing root length via the slow feedback loop created by the addition of the leaf  
307 model. Chemotropism effects from adding large amounts of P fertiliser could perhaps explain any  
308 differences between plant P uptake values at early growth stages. In the winter barley scenario, as  
309 root growth rate was dependent upon temperature, we see periods of no root growth matching  
310 periods of low temperature, as expected.

311 Heppell *et al.* (2015) considered the effects of discrete placing of fertiliser within the root zone  
312 against incorporating fertiliser throughout the soil for a range of cultivation options (mix 25, 20 and  
313 10 cm, inverted plough, minimum tillage and no cultivation) for winter barley at GS92. We do the  
314 same in this paper for the new coupled model (Figure 9). We arrive at the same overall conclusion,  
315 placing fertiliser rather than incorporating achieves a higher plant P uptake estimate and under a  
316 wet climate ([x5 flux of water at soil surface](#)), such as in the UK, this difference decreases (9.9% to 0.3%  
317 and 9.8% to 4.5%) over no cultivation for a dry and wet climate respectively. Ploughing was also the  
318 best cultivation option moving top soil P to a lower depth, making it more accessible to a  
319 comparatively larger root system.

## 320 Discussion

321 In order to obtain a more accurate representation of the growth of barley throughout a crop life  
322 cycle we have combined a below ground root-soil model with an above ground leaf model. By  
323 combining the two models we are able to let an above ground process (photosynthesis) affect a  
324 below ground process (root growth) and vice versa. C is created via photosynthesis in the leaf model  
325 (dependent upon [above ground dry leaf mass](#) and P) and stimulates root growth; increased root  
326 growth increases plant P uptake and hence [above ground dry leaf mass](#). This positive feedback effect  
327 could explain why crops with early plant P uptake levels grow more vigorously and can produce  
328 higher yields (Brenchley, 1929; Boatwright and Viets, 1966; Green *et al.*, 1973; Grant *et al.*, 2001).  
329 Due to possible unfavourable (e.g. dry) weather conditions, maximising early plant P uptake through  
330 greater root proliferation is also a good strategy to help ensure continuing capture of soil resources  
331 at later stages of growth.

332 From the modelling work conducted we can postulate that the whole crop model accurately  
333 estimates [above ground dry mass leaf mass](#) at all growth stages given it has accurate estimates of  
334 plant P uptake (an average difference of 4.6% for the whole crop model for [above ground dry leaf](#)  
335 mass, compared to 15.8% when using values one standard deviation away from the experimental  
336 data). Using the calibrated whole crop model we found the optimal fertiliser and cultivation scenario  
337 is to use a plough and place the P fertiliser. The largest increase in plant P uptake when placing  
338 fertiliser over incorporating fertiliser was 9.6% (plough, dry climate). The difference between  
339 incorporating and placing has been long studied and depends upon a range of criteria such as soil P  
340 concentration, soil temperature, crop species and price (Devine *et al.*, 1964; Mahler, 2001). Owusu-  
341 Gyimah *et al.* (2013) found that applying fertiliser at a depth of 10 cm and 20 cm away from the  
342 plant (placed P) gave the best outcome for maize growing under tropical conditions. By placing  
343 fertiliser instead of incorporating it throughout the soil the available P is being put where the root  
344 system is going to grow hoping to ensure early plant P uptake and a more successful crop. Hence



345 Wager et al. (1986) found that P fertilizer application rates could be halved by placing fertiliser  
346 instead of incorporation because the applied P was more efficiently used. However, optimal fertiliser  
347 and cultivation methods depend on the initial soil P condition/distribution (Randall and Hoeft, 1988);  
348 this includes at the depth at which existing P is initially available within the soil (Heppell *et al.*, 2015).

349 For modelling across countries it will be important to measure soil available P levels consistently, by  
350 either using a common method or a set of common descriptors. Although, an international ‘standard’  
351 soil extraction method is not necessarily needed; rather employing a basic soil property (e.g.  
352 sorption/buffer capacity) would be better to calibrate fertiliser recommendations. Modelling is the  
353 most appropriate way to overcome the problems of site specificity in soil P supply that confound  
354 current soil P test methods which do not apply to all soil types, i.e. across countries. Countries  
355 generally adopt a particular standard method for soil P tests; many different extractants are used.  
356 However, these do not necessarily give correlated results, for example across European laboratories  
357 (Neyroud and Lischer, 2002; Jordan-Meille *et al.*, 2012). It is possible that a more robust soil test will  
358 be developed in the future, that more accurately reflects immediate P availability to roots across  
359 different soil types. For example, using Diffusive Gradient in Thin films (DGT) based on soil P  
360 diffusion rates (Van Rotterdam *et al.*, 2009; Tandy *et al.*, 2011) or a method that mimics root P  
361 acquisition traits (De Luca *et al.*, 2015). The use of more mechanistic approaches to calculate soil  
362 available P levels via a more standardised test, or a combination of tests, enhances their applicability  
363 across a wider variety of soil types and may lead to more accurate assessment of fertiliser needs  
364 (Van Rotterdam *et al.*, 2014). Also, given that patterns of P concentration with depth in soil profiles  
365 vary between sites (Jobbágy and Jackson, 2001), it may also be important to assess surface  
366 stratification in no-tilled soils or in subsoils. Over-fertilising soils due to inaccurate estimation of  
367 requirement, or mis-interpretation of soil P supply through inappropriate tests leads not only to  
368 waste of finite reserves of phosphate-rock but also increased risk of P loss to water causing  
369 eutrophication (Hooda *et al.*, 2001). By using knowledge about the distribution of P within the soil  
370 and by modelling its implications, it should be possible to save on fertiliser costs by implementing

371 better optimised treatments through targeting P use (Yang *et al.*, 2013; Withers *et al.*, 2014).  
372 Furthermore, since crop and fertiliser management have long-term effects on topsoil and subsoil P  
373 availability (Bolland and Baker, 1998), it will be important to validate the model over several years if  
374 it is to improve on current simpler approaches to decision making. Additional model features would  
375 be needed, such as effects between cropping seasons, but would make for a more overall  
376 accomplished model. We note that the model would have to be calibrated separately for different  
377 crops.

378 Although there was little response to P application observed in the field trial in terms of plant P  
379 uptake at late growth stages (GS91 for spring barley and GS92 for winter barley), there was a  
380 response at early growth stages (GS31 for spring barley and GS39 for winter barley). This early  
381 response could imply that there were limiting environmental factors beyond nutritional inputs. Cold  
382 and dry conditions in spring are known to inhibit the transport of P from the soil to the root (Grant  
383 *et al.*, 2001). However, if the measured 'low' P soil was an underestimation for the total amount of  
384 available P in the soil then this could explain the lack of response at harvest observed in the field. In  
385 addition, field variation could in part explain the early response to applied P; however as the root  
386 system became larger during the latter growth stages any difference in plant P uptake and resulting  
387 yield was evened out. Due to the complex nature of cereal physiology (Sylvester-Bradley *et al.*, 2008),  
388 an early plant P uptake response does not necessarily indicate a higher final plant P uptake and yield;  
389 because the plant compensates by taking up more P later on as temperatures warm up. The slow  
390 feedback effect is a good explanation of the long term behaviour of the crop, and estimation of total  
391 plant P uptake.

392 Potentially, new ways to improve efficiency use of P can now be developed by combining recent  
393 advances in application technology, sensing technology, geo-spatial information and modelling so as  
394 to apply P where it is needed and importantly not apply it where it is not needed. Precision farming  
395 equipment is being widely adopted; now, its effective deployment depends on whether the vast

amount of data available about a given plot of land can be interpreted to improve the precision and decrease the risks compared to current decision making (Sylvester-Bradley *et al.*, 1999). For example, soil nutrient maps, past yield maps, soil and canopy sensors and climate predictions may provide input data for integrated crop models to output quantitative predictions of fertiliser requirements so that application as sowing can be adjusted in real time. However, the more immediate and preliminary prospect is of using simulation models to compare scenarios of possible treatments, to help guide future soil and fertiliser management strategies, and to accompany continuing field testing.

#### Acknowledgements

We would like to thank the BBSRC and DEFRA (BB/I024283/1) for funding S.P. and The Royal Society University Research Fellowship for funding T.R. K.C.Z. was partially funded by Award No. KUK-C1-13-04 of the King Abdullah University of Science and Technology (KAUST); J.H. by EPSRC Postdoctoral Prize Fellowship; and S.P., P.T., D.L., R.S-B., R.W., D.L.J. and T.R. by DEFRA, BBSRC, Scottish Government, AHDB, and other industry partners through Sustainable Arable LINK Project LK09136.

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574 **List of Tables**

575 Table 1: A list of the parameters used for the 3 models: leaf, root and coupled.

Parameter	Definition	Value	Units
<b>Leaf Model (Values from Thornley, J. H., 1995)</b>			
$k_G$	Leaf growth rate constant	1000	$\left(\frac{\text{kg C}}{\text{kg Leaf}} \frac{\text{kg P}}{\text{kg Leaf}} \text{day}\right)^{-1}$
$k_{litt}$	Leaf litter rate constant	0.05	$\text{day}^{-1}$
$k_{mlitt}$	Leaf litter Michealis-Menten constant	0.5	kg Leaf
$k_C$	Photosynthesis constant	0.1	$\frac{\text{kg C}}{\text{kg Leaf}} \text{day}^{-1}$
$k_M$	Leaf self-shading constant	1	kg Leaf
$J_C$	Carbon product inhibition constant	0.1	$\frac{\text{kg C}}{\text{kg Leaf}}$
$f_c$	Fraction of C used for leaf growth	(fitted)	$\frac{\text{kg C}}{\text{kg Leaf}}$
$f_p$	Fraction of P used for leaf growth	(fitted)	$\frac{\text{kg P}}{\text{kg Leaf}}$
$k_1$	P used for photosynthesis	(fitted)	$\frac{\text{kg Leaf}}{\text{kg P}}$
$k_p$	P:C ratio for photosynthesis production	0.005-0.05	$\frac{\text{kg P}}{\text{kg C}}$
$\beta_p$	Rate of P output to phloem	0	$\frac{\text{kg Leaf}}{\text{day}}$
$F$	Rate of P entry from xylem	Taken from barley experimental data or root model output	$\frac{\text{kgP}}{\text{day}}$
$A_T$	Air temperature	Taken from Local	$^{\circ}\text{C}$

		Met office MIDAS stations	
$\beta_c$	Rate of C output to phloem	(fitted)	$\frac{\text{kgL}}{\text{day}}$
$s_1$	Air temperature slope constant	(fitted)	-
$s_2$	Air temperature transition constant	(fitted)	$^{\circ}\text{C}$
<b>Root-Soil Model (values from Heppell <i>et al.</i>, 2015)</b>			
$D_0$	Water diffusivity	$10^3$	$\text{cm}^2 \text{day}^{-1}$
$K_s$	Water hydraulic conductivity	5	$\text{cm}^2 \text{day}^{-1}$
$D_f$	P diffusivity in free water	$10^{-5}$	$\text{cm}^2 \text{day}^{-1}$
$d$	Impedance factor	2	-
$a$	Main order root radius	0.085	cm
$a_1$	first order root radius	0.060	cm
$k_r$	Root radial water conductivity	$7.85 \cdot 10^{-6}$	$\text{m}^2 \text{s}^{-1} \text{MPa}^{-1}$
$k_z$	Root axial hydraulic conductivity	$1.198 \cdot 10^{-2}$	$\text{m}^4 \text{Pa}^{-1} \text{s}^{-1}$
$\psi_1$	Density of first order roots	2.33	$\text{cm}^{-1}$
$p_r$	Root internal xylem pressure	1	Pa
$p_c$	Characteristic suction pressure	0.0232	MPa
$L_0$	Max length of main order root	150	cm
$L_1$	Max length of first order root	7.9	cm
$L$	Root tip position	$0-L_0$	cm
$b$	Buffer power	23.28	-
$\theta$	Angle between the main root and first order branches	60	degrees
$\phi$	Porosity of soil	0.3	-
$p_r^0$	Initial root internal xylem pressure	1	Pa

$\lambda_1$	Root internal xylem pressure parameter	$2.7 \cdot 10^{-3}$	Pa/ degC
$\lambda_2$	Root internal xylem pressure parameter	$8.46 \cdot 10^{-4}$	Pa/% humidity
$\lambda_3$	Root internal xylem pressure parameter	$7.9 \cdot 10^{-2}$	Pa
$\delta$	Flux of water parameter	$2.69 \cdot 10^{-2}$	-
$\alpha$	Flux of water parameter	$1.2 \cdot 10^{-6}$	$\text{m s}^{-1}$ of water
$\beta$	Flux of water parameter	$2.22 \cdot 10^{-6}$	$\text{m s}^{-1}$ of water/degC
$\gamma$	Flux of water parameter	$5.35 \cdot 10^{-4}$	$\text{m s}^{-1}$ of water/ $\text{m s}^{-1}$ of air
$E$	Flux of water parameter	$5 \cdot 10^{-4}$	$\text{m s}^{-1}$ of water
$l_w$	Bottom of the soil	200	cm
<b>Coupled Model</b>			
$\gamma_c$	Root carbon growth parameter	(fitted)	-
$\alpha_c$	Strength of carbon effect on root growth	(fitted)	-
$A$	Strength of temperature dependence on root growth rate	0.0780	-

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577 **Table 2: The fitted parameter set for the leaf and coupled models, for spring barley and winter**  
578 **barley.**

Parameter		Value for Spring barley	Value for winter barley
Leaf Model	$\beta_c$	0.0001	0.0001

	$k_1$	100	859
	$f_c$	0.5	0.5
	$f_p$	$7 \cdot 10^{-4}$	$1.6 \cdot 10^{-3}$
	$s_1$	n/a	20.78
	$s_2$	n/a	-1.446
Coupled model	$\gamma_c$	$1.30 \cdot 10^{-5}$	$1.31 \cdot 10^{-4}$
	$\alpha_c$	1	1.982